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Heat transfer from a porous composite sphere immersed in a moving stream

JACOB H. MASLIYAH
Department of Chemical Engineering, University of Alberta, Edmonton, Alberta, Canada

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Abstract—Convection heat transfer in the low Reynolds number regions from an isothermal sphere surrounded by a porous shell is numerically evaluated. In the limit of large porous medium Peclet numbers, the average Nusselt number based on the porous medium thermal conductivity becomes independent of the fluid Peclet number and becomes proportional to the porous medium Peclet number raised to the power one-third. In the limit of small fluid and porous medium Peclet numbers, the average Nusselt number approaches that given by the limiting case of pure conduction.

INTRODUCTION

MIXED convection heat transfer from bodies embedded in a porous medium and for bodies immersed in a free moving stream have been studied quite extensively [1,2]. However, there is a small body of literature that deals with fluid flow and convective heat transfer for porous bodies immersed in a free moving stream. Fluid flow past a porous sphere has been studied by Brinkman [3], Ooms *et al.* [4], Neale *et al.* [5], Nandakumar and Masliyah [6] and that for a porous cylinder by Shi and Braden [7]. Convective heat transfer in the low Reynolds number regime has been reported by Ramilison and Gebhart [8]. No similar analysis is available for a porous sphere or other geometries.

In this study, the convective heat transfer in creeping flow will be examined for the case of an isolated composite sphere. Such a sphere comprises of a solid impermeable isothermal core surrounded by a shell of homogeneous and isotropic porous medium. The porous shell can represent either a medium having a low thermal conductivity that acts as an insulator or a medium having a high thermal conductivity which has the potential of enhancing heat transfer. The effects of the porous shell permeability, its thickness and its thermal conductivity will be studied. It should be recognized, that at low Reynolds numbers, natural convection is normally not insignificant especially for low Peclet numbers. In this study, the natural convection will be neglected and consequently this study deals with pure forced convection only. The limiting cases of low and high Peclet numbers for forced convection flow will be presented.

FORMULATION

The physical problem under study is shown in Fig. 1. The isolated composite sphere has an outer radius b and an inner solid isothermal spherical core of radius a. The inner core is surrounded by a shell of homogeneous and isotropic porous material of permeability K. The case of axisymmetric creeping flow of a Newtonian fluid having constant physical properties will be considered. The viscous dissipation, pressure, energy and buoyancy terms are neglected.

Within the unobstructed fluid outside the composite sphere, the Stokes and continuity equations describe the prevailing flow field and they are given by

$$\mu_f \nabla^2 \mathbf{u} = \nabla p, \quad b \leqslant r < \infty \tag{1}$$

and

$$\nabla \cdot \mathbf{u} = 0, \quad b \leqslant r < \infty \tag{2}$$

where $\mathbf{u} = [u_r, u_\theta, u_\phi]$ denotes the fluid velocity vector, and p is the fluid pressure. The corresponding equations describing the flow field within the permeable shell region are given by Brinkman's and continuity equations

$$-\frac{\mu_f}{K}\mathbf{u}^* + \mu_f \nabla^2 \mathbf{u}^* = \nabla p^*, \quad a \leqslant r \leqslant b \tag{3}$$

and

$$\nabla \cdot \mathbf{u}^* = 0, \quad a \leqslant r \leqslant b \tag{4}$$

where * denotes a macroscopically averaged quantity pertaining specifically to the porous medium region. The Brinkman equation was used in preference to the Darcy equation in order to accommodate the boundary conditions between the free fluid and the porous medium. Full discussions on the merits of the Brinkman equation can be found in Brinkman [3], Ooms *et al.* [4] and Koplik *et al.* [9].

The boundary conditions for equations (1)–(4) are those of no slip at the inner core, uniform flow at a distance far away from the sphere and continuity of velocities, normal and tangential stresses at the interface between the free surface and the fluid, $r = b$. The solution of flow equations (1)–(4) is given by Masliyah

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NOMENCLATURE

a	inner core (sphere) radius
b	composite sphere outer radius
C_{p_f}	fluid specific heat
h	local heat transfer coefficient
k_f	fluid thermal conductivity
k_m	effective thermal conductivity of porous medium
K	porous medium permeability
Nu_f	local Nusselt number based on k_f
Nu_m	local Nusselt number based on k_m
p	pressure
Pe_f	fluid Peclet number, $(2aU_\infty\rho_f/\mu_f)(C_{p_f}\mu_f/k_f)$
Pe_m	effective porous medium Peclet number, $(2aU_\infty\rho_f/\mu_f)(C_{p_f}\mu_f/k_m)$
r	radial coordinate
T	temperature
T_w	wall temperature
T_∞	fluid temperature far away from the sphere
$\mathbf{u}$	velocity vector
u_r	radial velocity
u_θ	angular velocity
U_r	dimensionless radial velocity, u_r/U_∞
U_θ	dimensionless angular velocity, u_θ/U_∞
U_∞	free stream approach velocity.

Greek symbols

α	dimensionless solid sphere radius, $a/\sqrt{K}$
β	dimensionless outer shell radius, $b/\sqrt{K}$
θ	angular coordinate
μ_f	fluid viscosity
ξ	dimensionless radial coordinate, $r/\sqrt{K}$
ρ_f	fluid density
ϕ	dimensionless temperature
ψ	stream function.

Subscripts

f	fluid
m	porous medium
r	radial direction
θ	angular direction.

Superscripts

*	pertaining to porous medium
—	average quantity.

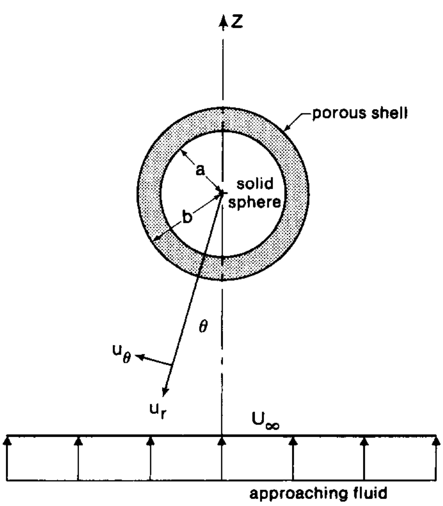

FIG. 1. Coordinate system for axisymmetric flow relative to an isolated composite sphere.

et al. [10] in terms of stream functions as

$$\psi = -\frac{KU_\infty}{2}[A/\xi+B\xi+C\xi^2+D\xi^4]\sin^2\theta, \quad \beta \leqslant \xi < \infty \quad (5)$$

$$\psi^* = -\frac{KU_\infty}{2}\left[E/\xi+F\xi^2+G\left(\frac{\cosh\xi}{\xi}-\sinh\xi\right)\right.$$
$$\left.+H\left(\frac{\sinh\xi}{\xi}-\cosh\xi\right)\right]\sin^2\theta, \quad \alpha \leqslant \xi \leqslant \beta \quad (6)$$

where

$$\xi = r/\sqrt{K}, \quad \alpha = a/\sqrt{K} \quad \text{and} \quad \beta = b/\sqrt{K}.$$

The stream functions are related to the velocities by

$$u_r = \frac{-1}{r^2\sin\theta}\frac{\partial\psi}{\partial\theta} \quad (7)$$

$$u_\theta = \frac{1}{r^2\sin\theta}\frac{\partial\psi}{\partial r}. \quad (8)$$

Similarly u_r^* and u_θ^* are related to ψ^*. The constants in equations (5) and (6) are given in the appendix.

The energy equation for the free fluid and porous medium are given by

$$\rho_f C_{p_f}\left(u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{r}\frac{\partial T}{\partial\theta}\right) = k_f\nabla^2 T \quad (9)$$

and

$$\rho_f C_{p_f}\left(u_r^*\frac{\partial T^*}{\partial r}+\frac{u_\theta^*}{r}\frac{\partial T^*}{\partial\theta}\right) = k_m\nabla^2 T^*. \quad (10)$$

Introducing the dimensionless quantities

$$\xi = r/\sqrt{K}, \quad U_r = u_r/U_\infty, \quad U_\theta = u_\theta/U_\infty$$

$$U_r^* = u_r^*/U_\infty, \quad U_\theta^* = u_\theta^*/U_\infty, \quad \phi = \frac{T-T_\infty}{T_w-T_\infty}$$

$$\phi^* = \frac{T^*-T_\infty}{T_w-T_\infty}, \quad Pe_m = \frac{2aU_\infty \rho_f C_{p_f}}{k_m}$$

and

$$Pe_f = \frac{2aU_\infty \rho_f C_{p_f}}{k_f}. \tag{11}$$

The energy equations become

$$U_r\frac{\partial \phi}{\partial \xi} + \frac{U_\theta}{\xi}\frac{\partial \phi}{\partial \theta} = \frac{2\alpha}{Pe_f}\left[\frac{1}{\xi^2}\frac{\partial}{\partial \xi}\left(\xi^2\frac{\partial \phi}{\partial \xi}\right) + \frac{1}{\xi^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial \phi}{\partial \theta}\right)\right] \tag{12}$$

and

$$U_r^*\frac{\partial \phi^*}{\partial \xi} + \frac{U_\theta^*}{\xi}\frac{\partial \phi^*}{\partial \theta} = \frac{2\alpha}{Pe_m}\left[\frac{1}{\xi^2}\frac{\partial}{\partial \xi}\left(\xi^2\frac{\partial \phi^*}{\partial \xi}\right) + \frac{1}{\xi^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial \phi^*}{\partial \xi}\right)\right]. \tag{13}$$

The boundary conditions are

$$r = a, \quad \xi = \alpha, \quad \phi^* = 1 \quad \text{for all } \theta \tag{14a}$$

$$r = b, \quad \xi = \beta, \quad \phi = \phi^* \quad \text{for all } \theta \tag{14b}$$

$$r = b, \quad \xi = \beta, \quad Pe_f\frac{\partial \phi^*}{\partial \xi} = Pe_m\frac{\partial \phi}{\partial \xi} \quad \text{for all } \theta \tag{14c}$$

$$r \to \infty, \quad \xi \to \infty, \quad \phi = 0 \quad \text{for all } \theta \tag{14d}$$

$$\theta = 0 \text{ and } \pi, \quad \frac{\partial \phi}{\partial \theta} = \frac{\partial \phi^*}{\partial \theta} = 0 \quad \text{for all } \xi. \tag{14e}$$

The velocity fields are provided from the analytical solution. Equations (12) and (13) were numerically solved using a control volume approach [11].

The heat transfer coefficient is given by

$$h = -\frac{k_m}{(T_w-T_\infty)}\frac{\partial T^*}{\partial r}\bigg|_{r=a}. \tag{15}$$

Introducing dimensionless quantities equation (15) becomes

$$h = -\frac{k_m}{\sqrt{K}}\frac{\partial \phi^*}{\partial \xi}\bigg|_{\xi=\alpha}. \tag{16}$$

Two types of Nusselt numbers can be defined, one based on k_f and another based on k_m, i.e.

$$Nu_f = \frac{2ah}{k_f}$$

and

$$Nu_m = \frac{2ah}{k_m}.$$

Making use of equation (16), one can obtain the local Nusselt numbers

$$Nu_f = -2\alpha\frac{Pe_f}{Pe_m}\frac{\partial \phi^*}{\partial \xi}\bigg|_{\xi=\alpha} \tag{17}$$

and

$$Nu_m = -2\alpha\frac{\partial \phi^*}{\partial \xi}\bigg|_{\xi=\alpha}. \tag{18}$$

The average Nusselt number is given by

$$\overline{Nu} = \frac{1}{2}\int_0^\pi Nu \sin\theta \, d\theta. \tag{19}$$

In the limiting case of pure conduction, the diffusion equations can be simultaneously solved for composite spherical shells using the boundary conditions of equations (14a)–(14d). The average Nusselt number for pure conduction is then given by

$$\overline{Nu}_m = \frac{2}{1+(k_m/k_f-1)(\alpha/\beta)}. \tag{20a}$$

Putting $k_m/k_f \equiv Pe_f/Pe_m$ equation (20a) becomes

$$\overline{Nu}_m = \frac{2}{1+(Pe_f/Pe_m-1)(\alpha/\beta)} \tag{20b}$$

where

$$(\alpha/\beta) < 1.$$

Here k_m refers to the spherical shell bounded by $\alpha \leqslant \xi < \beta$ and k_f refers to the spherical shell $\xi \geqslant \beta$. In the limit of $Pe_f = Pe_m$, the average Nusselt number becomes 2 which is the accepted Nusselt number for a sphere in pure conduction.

NUMERICAL PROCEDURE

Initially, the convective problem was formulated using central difference formulation. However, it was abandoned due to convergence problems at high transport numbers. The problem was reformulated using the control volume approach as described by Patankar [11]. In the porous region, the grid size was uniform. The number of grids in the radial direction was varied from 11 to 151 depending on the values of α and β. The highest number of grids was used for the case of $\alpha = 0.1$, $\beta = 0.3$ at $Pe_m > 1000$. In the free fluid zone, the grid size was varied as

$$(\Delta\xi)_{i+1} = 1.12^{(i-1)}(\Delta\xi)_i$$

where $(\Delta\xi)_i$ is the grid size at the ith radial node with $i = 1$ when $\xi = \beta$.

The number of grids in the free fluid region was varied to ensure that the fluid temperature at the downstream region ($\theta = \pi$) decayed to zero monotonically. The grid size in the angular region was taken as $\pi/20$. Tests were conducted with smaller grid sizes and it was found that the results for the Nusselt number were not affected by more than 0.5%. Results for a bare sphere with $Pe_m = Pe_f$ were very close to those given by Acrivos and Goddard [12]

$$Nu = 0.991 Pe^{1/3} + 0.992.$$

Due to round-off errors, it was necessary to evaluate the velocity field from the analytical solution using double precision arithmetic. However, the calculations for the heat transfer were made using single precision arithmetic. The finite-difference equations were solved iteratively using an FPS-164 array processor.

DISCUSSION OF RESULTS

The flow and energy equations indicate that there are four parameters that govern the heat transfer from a composite sphere. These are α, β, Pe_f and Pe_m. α is a measure of the ratio of the inner core radius to the square root of the shell permeability, K. Similarly, β is a measure of the composite sphere outer radius. The ratio β/α can be taken to represent the radii ratio b/a. It is best to think of α in terms of a fixed inner core radius and its variation is due to changes in the shell permeability.

Four cases are considered for a wide range of Peclet numbers. They are: (i) $\alpha = 0.1$, $\beta = 0.11$; (ii) $\alpha = 0.1$, $\beta = 0.3$; (iii) $\alpha = 5.0$, $\beta = 5.5$; and (iv) $\alpha = 5$, $\beta = 15$. Cases (i) and (ii) represent a shell having high permeability whereas cases (iii) and (iv) are for a low permeability shell. Cases (i) and (iii) have a small shell thickness whereas cases (ii) and (iv) have a thick porous shell.

Figure 2 shows the stream function contours for the four cases cited above. For the cases of $\alpha = 0.1$, $\beta = 0.11$ and 0.3, and $\alpha = 5$, $\beta = 5.5$, the streamlines are very similar. Moreover, they are similar to the case of flow past a bare sphere of radius a. For the case of $\alpha = 5$, $\beta = 15$, the streamlines pronounced upward shift away from the line of symmetry clearly indicates that the resistance to flow within the porous shell is high and that there is a substantial decrease in the flow through the porous medium.

The temperature variations along $\theta = 0$ (frontal stagnation line) are shown for $\alpha = 0.1$, $\beta = 0.11$ and for $\alpha = 0.1$, $\beta = 0.3$ in Figs. 3 and 4, respectively. Figure 3 shows that for $Pe_m = 10{,}000$ the temperature variation is independent of the fluid Peclet number and that the thermal boundary layer is within the porous shell. However, for the case of $Pe_m = 100$, the temperature variation is a strong function of the fluid Peclet number, Pe_f. For the case of (Pe_m, Pe_f) of (100, 1), (100, 10) and (100, 1000) there is a sharp change in the temperature profile at the edge of the composite sphere ($\xi = \beta$). For the case of $\alpha = 0.1$ and $\beta = 0.3$, Fig. 4 shows that the thermal boundary layer is within the porous shell for a Pe_m as low as 10. Figures 3 and 4 suggest that in cases where the thermal boundary layers lie within the porous medium, it is expected that the Nusselt number, $\overline{Nu}_m$, becomes independent of the fluid Peclet number, Pe_f. When the

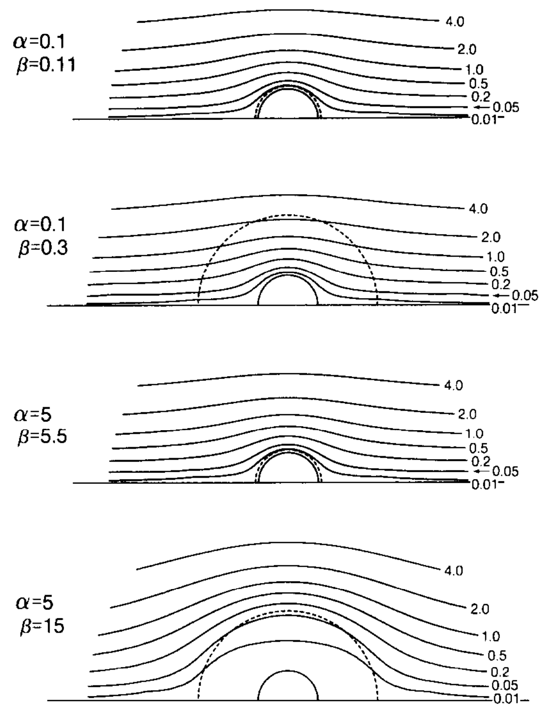

FIG. 2. Streamline contours.

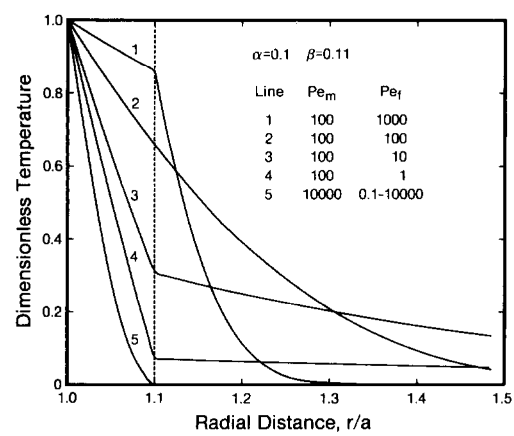

FIG. 3. Variation of temperatures with radial distance for $\alpha = 0.1$, $\beta = 0.11$.

porous shell thickness is large, such an independence of Pe_f should then occur at a smaller value of the porous medium Peclet number.

The variation of the average Nusselt number, $\overline{Nu}_m$, with the porous medium Peclet number is shown in Figs. 5 and 6. For the cases of $\alpha = 0.1$, $\beta = 0.11$ at a porous medium Peclet number of 20,000 the average Nusselt number, $\overline{Nu}_m$, becomes independent of the fluid Peclet number. Such an asymptote is reached at a much lower value of Pe_m for the case of $\alpha = 0.1$, $\beta = 0.3$ as would have been predicted from the temperature profiles of Figs. 3 and 4. For both cases, at high values of Pe_m, Nusselt number variation with Pe_m becomes coincidental with that of a bare sphere where $Pe_f = Pe_m$. In this regime, $\overline{Nu}_m \simeq 0.99 Pe_m^{1/3}$. Figure 6 shows the Nusselt number variation, $\overline{Nu}_m$,

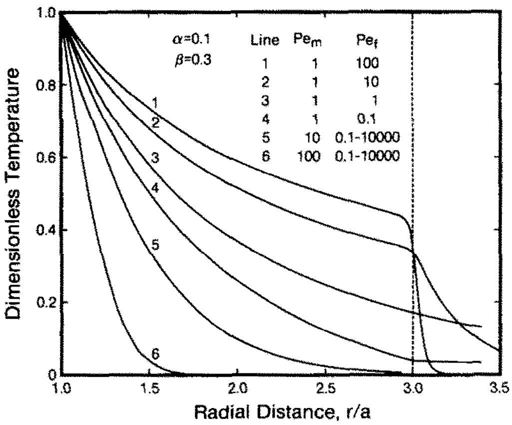

FIG. 4. Variation of temperatures with radial distance for $\alpha = 0.1$, $\beta = 0.3$.

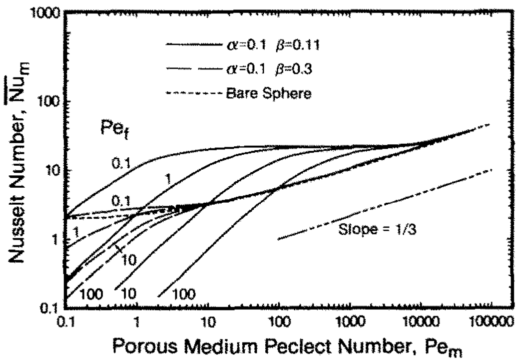

FIG. 5. Variation of average Nusselt number with porous medium Peclet number for the high permeability case.

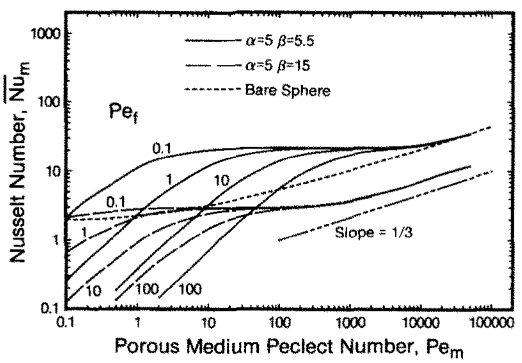

FIG. 6. Variation of average Nusselt number with porous medium Peclet number for the low permeability case.

with Pe_m for the cases of $\alpha = 5$, $\beta = 5.5$ and $\alpha = 5$, $\beta = 15$. The variation of $\overline{Nu}_m$ for $\alpha = 5$, $\beta = 5.5$ is very similar to that of $\alpha = 0.1$, $\beta = 0.11$. This is due to the fact that the flow field is little affected by the porous shell which is quite thin for these two cases. For $\alpha = 5$ and $\beta = 15$, $\overline{Nu}_m$ becomes invariant to Pe_f at a Pe_m value of about 100 and it becomes proportional to $Pe_m^{1/3}$. However, it falls below the bare sphere ($Pe_f = Pe_m$) case. This is because the velocity in the porous medium is much less than that with the absence of the porous shell.

In order to assess the effect of the presence of the

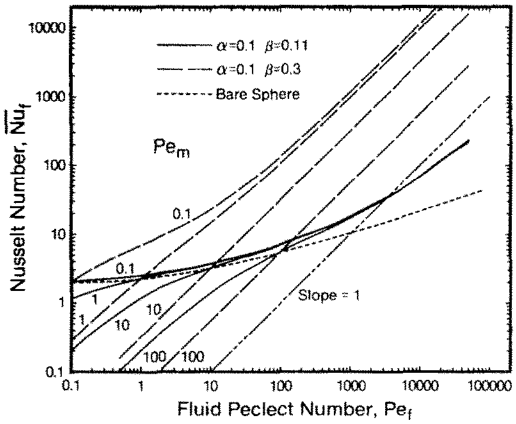

FIG. 7. Variation of average Nusselt number with fluid Peclet number for the high permeability case.

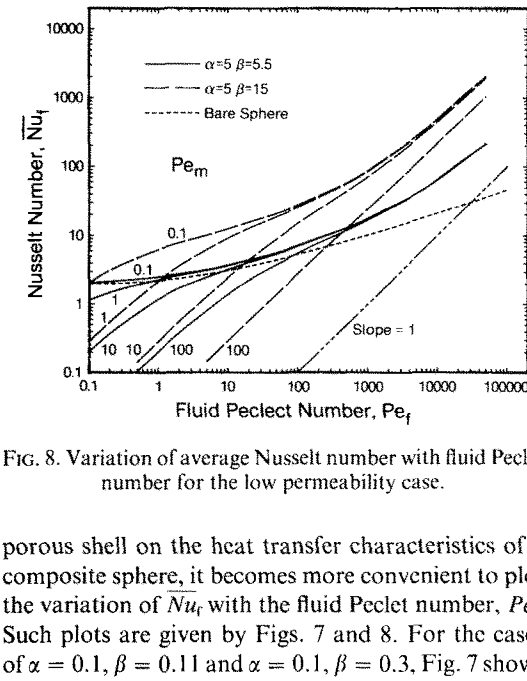

FIG. 8. Variation of average Nusselt number with fluid Peclet number for the low permeability case.

porous shell on the heat transfer characteristics of a composite sphere, it becomes more convenient to plot the variation of $\overline{Nu}_f$ with the fluid Peclet number, Pe_f. Such plots are given by Figs. 7 and 8. For the cases of $\alpha = 0.1$, $\beta = 0.11$ and $\alpha = 0.1$, $\beta = 0.3$, Fig. 7 shows that for a given value of Pe_f, enhancement in heat transfer can be achieved as long as $Pe_m < Pe_f$. Heat transfer enhancement is very pronounced for the case of $\alpha = 0.1$, $\beta = 0.3$. It should be noted, that a $Pe_m < Pe_f$ is equivalent to $k_m > k_f$. Figure 7 shows that for a $Pe_f = Pe_m$, $\overline{Nu}_f$ for the composite sphere is very close to that of a bare sphere. This is because the flow resistance offered by the porous shell is fairly small at $\alpha = 0.1$. For the case of $\beta = 0.3$, in the range of high Pe_f, it can be observed that ln $\overline{Nu}_f$ varies linearly with ln Pe_f, and ln $\overline{Nu}_f$ vs ln Pe_f exhibits a slope of unity. This leads to

$$\overline{Nu}_f = (\text{constant})\ Pe_f$$

and

$$\frac{2ah}{k_f} = (\text{constant})\ Re \cdot \frac{C_{p_f}\mu_f}{k_f}$$

with h independent of the fluid thermal conductivity.

The Nusselt number $\overline{Nu}_f$ variation with Pe_f is shown

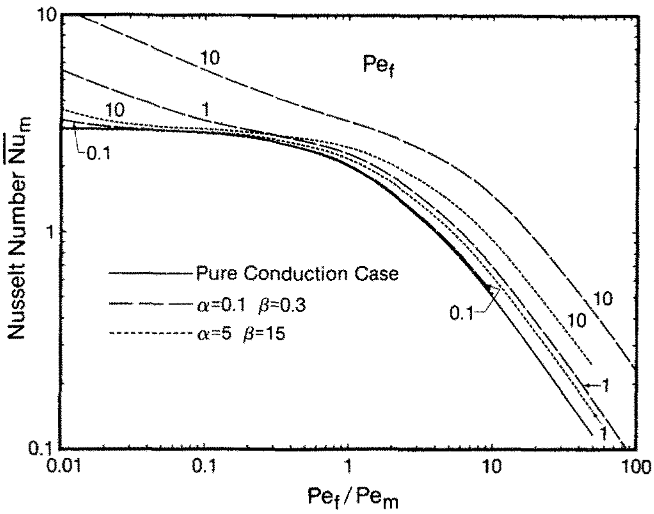

FIG. 9. Comparison with limiting case of pure conduction.

in Fig. 8. For the case of $\alpha = 5$, $\beta = 15$ with $Pe_f \geqslant 10$, and $Pe_f = Pe_m$, the $\overline{Nu}_f$ for the composite sphere is less than that of a bare sphere. This means that no enhancement in heat transfer is achieved until the Pe_m value is much less than Pe_f. Here, the retardation in the velocity field in the porous medium tends to lower the value of $\overline{Nu}_f$ until the Pe_m value is sufficiently lower than Pe_f. Consequently, if a porous shell is used to enhance heat transfer from a body, unless the porous medium thermal conductivity is sufficiently larger than that of the fluid, the porous shell might not offer any enhancement in heat transfer.

For pure conduction, equation (20b) describes the variation of $\overline{Nu}_m$ as a function of (α/β) and (Pe_f/Pe_m). It is of interest to evaluate the validity of equation (20b) for forced convection at small values of Pe_m and Pe_f. Figure 9 shows the variation of $\overline{Nu}_m$ with Pe_f/Pe_m as given by equation (20b) for the pure conduction case and for the force convection from this study. For $\alpha = 5$, $\beta = 15$ with $Pe_f = 0.1$ and 1.0, the variation of the average Nusselt number, $\overline{Nu}_m$, is fairly close to the pure conduction case. However, as Pe_f is increased to 10, deviation from the limiting case of the pure conduction becomes more significant. For $\alpha = 0.1$, $\beta = 0.3$ the agreement with the pure conduction case is close only for the smaller values of Pe_f (and consequently lower values of Pe_m). The closer agreement to the pure conduction limit for $\alpha = 5$ and $\beta = 15$ is due to the fact that for this case the porous medium is not very permeable and the velocity field is weak. This in turn decreases the effect of the presence of the external flow field.

It was stated earlier that buoyancy effects were neglected. If one is to include buoyancy forces, it becomes necessary to modify Brinkman's equation to account for the inertial effects that can arise from flow within a fairly permeable porous medium. In addition, the inertial terms that are absent in Stokes equations must also be accounted for by using Navier–Stokes equations.

CONCLUSIONS

The solution of the convection–diffusion equation showed that at high values of the porous medium Peclet number, the average Nusselt number, $\overline{Nu}_m$, becomes independent of the fluid Peclet number. In the region of small Pe_f and Pe_m, the analytical expression for pure conduction provides a good approximation to the average Nusselt number. The average Nusselt number is found to be a strong function of the porous shell permeability and thickness.

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REFERENCES

1. P. Cheng, *Handbook of Heat Transfer Fundamentals* (Edited by W. M. Rohsenow, J. Hartnett and E. N. Garic), 2nd edn. McGraw-Hill, New York (1985).
2. R. Clift, J. R. Grace and M. E. Weber, *Bubbles, Drops and Particles*. Academic Press, New York (1978).
3. H. C. Brinkman, A calculation of the viscous force exerted by a flowing fluid on a dense swarm of particles, *Appl. Scient. Res.* **A1**, 27–34 (1947).
4. G. Ooms, P. F. Mijnlieff and H. L. Beckers, Frictional force exerted by a flowing fluid on a permeable particle, with particular reference to polymer coils, *J. Chem. Phys.* **53**, 4123–4130 (1970).
5. G. Neale, N. Epstein and W. Nader, Creeping flow relative to permeable spheres, *Chem. Engng Sci.* **28**, 1865–1874 (1973). Erratum: **29**, 1352 (1974).
6. K. Nandakumar and J. H. Masliyah, Laminar flow past a permeable sphere, *Can. J. Chem. Engng* **60**, 202–211 (1982).
7. Y. Y. Shi and R. E. Braden, Jr., The effect of permeability on low Reynolds number flow past a circular porous cylinder. In *Develop. Theoret. Appl. Mech.*

(Edited by W. A. Shaw), Vol. 3, pp. 761–775. Pergamon Press, Oxford (1966).
8. J. M. Ramilison and B. Gebhart, Heat transfer from a porous circular cylinder immersed in a moving stream, *Int. J. Heat Mass Transfer* **25**, 1912–1916 (1982).
9. J. Koplik, H. Levine and A. Zee, Viscosity renormalization in the Brinkman equation, *Physics Fluids* **26**, 2864–2870 (1983).
10. J. H. Masliyah, G. Neale, K. Malysa and T. G. M. van de Ven, Creeping flow over a composite sphere: solid core with porous shell, *Chem. Engng Sci.* **42**, 245–253 (1987).
11. S. V. Patankar, *Numerical Heat Transfer and Fluid Flow*. McGraw-Hill, New York (1980).
12. A. Acrivos and J. D. Goddard, Asymptotic expansion for laminar forced-convection heat and mass transfer, *J. Fluid Mech.* **23**, 273–291 (1965).

APPENDIX

The constants appearing in equations (5) and (6) are given as

$$B = \frac{B_0}{2(\alpha \sinh \beta - \cosh \alpha)J}$$

where

$$B_0 = 3(\alpha^4 + 2\alpha\beta^3 + 3\alpha^2)\cosh\alpha + 9\alpha^2(\cosh\beta - \beta\sinh\beta - \alpha\sinh\alpha) + 3\cosh\Delta[(\alpha^3 + 2\beta^3 + 3\alpha)(\alpha\beta\sinh\beta - \alpha\cosh\beta - \beta\cosh\alpha) + 3\alpha^2\beta\sinh\alpha] + 3\sinh\Delta[(\alpha^3 + 2\beta^3 + 3\alpha)\cosh\alpha + 3\alpha^2(\alpha\beta\sinh\beta - \alpha\cosh\beta - \sinh\alpha)]$$

and

$$J = -6\alpha + (3\alpha + 3\beta + \alpha^3 + 2\beta^3)\cosh\Delta + 3(\alpha^2 - 1)\sinh\Delta$$

with

$$\Delta = \beta - \alpha$$

$$H = \frac{3(\alpha^3 + 2\beta^3)\cosh\alpha + 9\alpha(\cosh\alpha - \alpha\sinh\alpha - \cosh\beta + \beta\sinh\beta)}{J}$$

$$G = \frac{3\alpha - (\alpha\cosh\beta - \sinh\alpha)H}{\alpha\sinh\beta - \cosh\alpha}$$

$$F = (G\cosh\alpha + H\sinh\alpha)/3\alpha$$

$$E = 2B + 2\beta^3 F$$

$$D = 0$$

$$C = -1$$

$$A = \beta^3 - \beta^2 B + E + \beta^3 F + (\cosh\beta - \beta\sinh\beta)G + (\sinh\beta - \beta\cosh\beta)H.$$

TRANSFERT DE CHALEUR PAR UNE SPHERE COMPOSITE POREUSE IMMERGEE DANS UN ECOULEMENT

Résumé—On évalue numériquement le transfert convectif de chaleur, pour des nombres de Reynolds faibles, par une sphère isotherme entourée d'une couche poreuse. Dans le cadre des grands nombres de Peclet poreux, le nombre de Nusselt moyen, basé sur la conductivité thermique du milieu poreux, devient indépendant du nombre de Peclet du fluide et il est proportionnel au nombre de Peclet du milieu poreux élevé à la puissance un-tiers. Dans la limite des petits nombres de Peclet du fluide et du milieu poreux, le nombre de Nusselt moyen s'approche de celui correspondant au cas limite de la conduction pure.

WÄRMEÜBERGANG AN EINER KUGEL MIT PORÖSER SCHALE IN EINEM STRÖMENDEN FLUID

Zusammenfassung—Es wird der konvektive Wärmeübergang an einer isothermen Kugel mit poröser Schale bei niedrigen Reynolds-Zahlen numerisch berechnet. Im Grenzfall großer Peclet-Zahlen des porösen Stoffes wird die mittlere Nusselt-Zahl—gebildet mit der Wärmeleitfähigkeit des porösen Materials—von der Peclet-Zahl des Fluids unabhängig, sie steigt proportional zur Peclet-Zahl des porösen Materials mit der Potenz 1/3. Im Grenzfall kleiner Peclet-Zahlen des Fluids und mittlerer Peclet-Zahlen des porösen Stoffes nähert sich die mittlere Nusselt-Zahl derjenigen bei reiner Wärmeleitung.

ТЕПЛОПЕРЕНОС ОТ ПОРИСТОЙ КОМПОЗИТНОЙ СФЕРЫ, ПОГРУЖЕННОЙ В ДВИЖУЩИЙСЯ ПОТОК

Аннотация—Численно оценен конвективный теплоперенос в области низких чисел Рейнольдса от изотермической сферы, окруженной пористой оболочкой. В предельном случае для больших чисел Пекле среднее число Нуссельта, которое содержит коэффициент теплопроводности пористой среды, становится независящим от числа Пекле для жидкости и пропорциональным этому числу для пористой среды в степени 1/3. В пределе малых чисел Пекле для жидкости и пористой среды усредненное число Нуссельта стремится к значению, полученному для предельного случая кондуктивной теплопроводности.